TOPIC: VECTORS (A' LEVEL)

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"Revise, Reflect, Achieve"

- This leaflet contains well-researched about – mathematics vectors related questions.
- It's meant to boost confidence, enhance question interpretation and familiarize A' level Maths students with topic content.
- **JK** reserves the right to any error and comment in & about this leaflet.

REVISION QUESTIONS

1. In a triangle OAB, X is a point on OB such that $\overrightarrow{OX} = 2\overrightarrow{XB}$ and Y is a point on AB such that $2\overrightarrow{BY} = 3\overrightarrow{YA}$.

    i) Express $\overrightarrow{OX}$ and $\overrightarrow{OY}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ where $\boldsymbol{a} = \overrightarrow{OA}$ and $\boldsymbol{b} = \overrightarrow{OB}$.

    ii) Find the position vector of any point on XY, and hence, find the position vector of point Z, where $\overline{XY}$ produced meets $\overline{OA}$ produced.

    iii) Calculate the ratio of $\overrightarrow{AZ}:\overrightarrow{0Z}$

2. In a triangle OAB, $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$**.** A point L is on the side AB and M on the side OB. OL and AM meet at S. $\overline{AS} = \overline{SM}$ and $\overrightarrow{OS} = \frac{3}{4}\overrightarrow{OL}$. Given that $\overrightarrow{OM} = n\overrightarrow{OB}$ and $\overrightarrow{AL} = m\overrightarrow{AB}$. Express the vectors;

    a) AM and OS in terms of $\boldsymbol{a}, \boldsymbol{b}$ and n.

    b) OL and OS in terms of $\boldsymbol{a}, \boldsymbol{b}$ and m

    Hence, find the values of n and m.

3. In a triangle OPR, $\overrightarrow{OP} = \boldsymbol{p}$ and $\overrightarrow{OR} = \boldsymbol{r}$. Points A and B lie on $\overline{OP}$ and $\overline{RP}$ respectively such that $\overrightarrow{RB}:\overrightarrow{RP} = 3:7$ and $\overline{OP}:\overline{AP} = 3:1$. Lines AB and OR are both produced to meet at point M. find the position vector of M in terms of $\boldsymbol{r}$.

4. a) If $\boldsymbol{a} = 2\boldsymbol{i} + \lambda\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = \beta\boldsymbol{i} + 4\boldsymbol{j} - \frac{\gamma}{2}\boldsymbol{k}$ are equal, find the values of λ, β and γ.

   b) Given that the position vectors; $\boldsymbol{i} + \mu\boldsymbol{j} - 3\boldsymbol{k},\ 2\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $3\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ are co-planar, find the value of μ.

5. Given that; $\boldsymbol{m} = 2\boldsymbol{i} + \beta\boldsymbol{j},\ \boldsymbol{n} = \mu\boldsymbol{i} - 3\boldsymbol{j}$ and $\boldsymbol{c} = \boldsymbol{i} - \boldsymbol{j}$. Find the value of;
   i). β if $\boldsymbol{m}$ is perpendicular to $\boldsymbol{c}$.
   ii). μ if $\boldsymbol{n}$ is parallel to $\boldsymbol{c}$.

6. A quadrilateral ABC has vertices A, B and C with position vectors $\boldsymbol{-4i + 6j, 3i + 5j, 4i - 2j}$ and $\boldsymbol{-3i - j}$ respectively. Verify whether the quadrilateral is a rhombus or a parallelogram.

7. The vertices of a quadrilateral ABC are A(-3,2), B(1,-2), C(5,2) and D(1,6). Prove whether ABCD is a rectangle, rhombus, parallelogram or a square.

8. a) The position vectors of points A, B and C are $\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k},\ 2\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$ and $5\boldsymbol{i} - 2\boldsymbol{j} - 15\boldsymbol{k}$ respectively. Show that A, B and C are collinear. Hence deduce the ratio in which B divides $\overline{AC}$.

   b) Given that $\boldsymbol{a} = \alpha\boldsymbol{i} + (2 + \alpha)\boldsymbol{j} + 3\boldsymbol{k}$ and $\boldsymbol{b} = 3\boldsymbol{j} - \boldsymbol{i} + (4 - \alpha)\boldsymbol{k}$ are perpendicular to one another, find the value of α.

9. a) Using vectors, find the acute angle between lines; $2y = x + 3$ and $3y + 2x = 1$

   b) Given the position vectors $\boldsymbol{n} = \boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$ and $\boldsymbol{r} = -2\boldsymbol{i} + \boldsymbol{k}$, find the;
   i). a unit vector normal to both $\boldsymbol{n}$ and $\boldsymbol{r}$.
   ii). angle between vectors $\boldsymbol{n}$ and $\boldsymbol{r}$.

   (***In each case, Use both cross and dot product***)

10. The points A, B and C have position vectors $\boldsymbol{4i + 10j + 6k},\ \boldsymbol{6i + 8j - 2k}$ and $\boldsymbol{i + 10j + 3k}$ respectively.

    a) Show A, B and C are the vertices of a triangle

b) Prove that a triangle ABC is right angled.

c) Using vector product, find the size of $< ABC$

d) Solve the triangle ABC

11. Find the area of a rectangle whose adjacent sides are represented by position vectors $3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $5\boldsymbol{i} - \boldsymbol{j} - 8\boldsymbol{k}$.

12. The points A, B and C have position vectors $\mathbf{-2\boldsymbol{i} + 3\boldsymbol{j}}$, $\boldsymbol{i} - \mathbf{2\boldsymbol{j}}$, $\mathbf{8\boldsymbol{i} - 5\boldsymbol{j}}$ respectively.

a) Find the equation of line AC

b) Determine the co-ordinates of point D, if ABCD is a parallelogram

c) Write down the vector equation of the line through point B perpendicular to AC and find where it meets AC.

13. A point P divides points A and B whose position vectors are respectively; $\boldsymbol{j} - \boldsymbol{i} + 3\boldsymbol{k}$ and $-3\boldsymbol{k} + \boldsymbol{i}$ in the ratio 2:1. Find the **position vector** of P if it divides $\overline{AB}$;

a) Internally

b) Externally

14. The position vectors of points; A, B and C are $\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$, $2\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$ and $5\boldsymbol{i} - 2\boldsymbol{j} - 15\boldsymbol{k}$ respectively. Show that A, B and C are collinear. Hence, deduce the ratio in which B divides AC.

15. The line l has Equation: $\boldsymbol{r} = \boldsymbol{i} + 2\boldsymbol{j} + \mathbf{3}k + \gamma(2\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k})$. A point P has position vector $4\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$. Find the length of the perpendicular from P to l. (***Use both cross and dot product***)

16. The points A and B have position vectors $\boldsymbol{i} + \mathbf{2\boldsymbol{j}} + \mathbf{2\boldsymbol{k}}$ and $\mathbf{4\boldsymbol{i} + 3\boldsymbol{j}}$ respectively, relative to the origin O.

a) Find the lengths OA and OB

b) Find the scalar product of OA and OB. Hence, find angle AOB.

c) Find the area of triangle AOB, giving your answers correct to 3 dps.

d) The point C divides AB in the ratio $\beta : 1 - \beta$

i) Find the expression of $\overrightarrow{0C}$

ii) Show that $\left|\overrightarrow{OC}\right|^2 = 14\beta^2 + 2\beta + 9$.

iii) Find the position vectors of the two points on AB whose distance from O is $\sqrt{21}$

iv) Show that the shortest (perpendicular) distance of O from AB is approximately 2.99 units.

17. Three points P, Q and R have position vectors $\boldsymbol{7i + 10j}$, $\boldsymbol{3i + 12j}$ and $\boldsymbol{-i + 4j}$.

a) Write down the vectors $\overrightarrow{PQ}$ and $\overrightarrow{RQ}$, and show that they are perpendicular.

b) Using a scalar product or otherwise, find angle PRQ

c) Find the position vector of S, the midpoint of PR

d) Show that $\left|\overrightarrow{QS}\right| = \left|\overrightarrow{RS}\right|$. Using your previous results or otherwise, find angle PSQ

18. The cartesian equation of the line is given by; $\frac{2-x}{-3} = \frac{y-3}{1} = \frac{4-2z}{2}$. Find;

a) Direction vector of the line

b) Two points that lie on a line

c) Vector equation of the line that passes through a point with position vector $\boldsymbol{-k + 4j}$ and is parallel to the given line.

19. Find the co-ordinates of P_2 if $2\mathrm{P_0P_1} = 3\mathrm{P_1P_2}$ where $\mathrm{P_0}(-2,7,4)$ and $\mathrm{P_1}(7,-2,1)$.

20. The lines l_1 and l_2 have vector equations; $\boldsymbol{r}_1 = 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{4}k + \alpha(\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k})$ and $\boldsymbol{r}_2 = -2\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k} + \beta(-2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$ respectively.

a) Show that l_1 and l_2 are skew.

b) Hence, find the distance between l_1 and l_2

c) The point P lies on l_1 and point M has a position vector $2\boldsymbol{i} - \boldsymbol{k}$.

i). Given that the line PM is perpendicular to l_1, find the position vector of P. Hence, write the cartesian Equation of line PM

ii). Verify that M lies on l_2, and that PM is perpendicular to l_2.

21. With respect to the origin O, the points A and B have position vectors given by $\overrightarrow{OA} = 2\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ and $\overrightarrow{OB} = \boldsymbol{i} + 4\boldsymbol{j} + 3\boldsymbol{k}$. The line l has vector equation: $\boldsymbol{r} = 4\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k} + \beta(\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k})$.

a) Prove that l does not intersect the line through A and B.

b) Hence, find the shortest distance between l and line AB.

22. Relative to the origin O, the position vectors of P and Q are given by $\overrightarrow{OP} = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ and $\overrightarrow{OQ} = 4\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$.

    a) Use vector product to find angle POQ, correct to the nearest degree.

    b) Find the unit vector in the direction of $\overrightarrow{PQ}$.

    c) The point R is such that $\overrightarrow{OR} = 6\boldsymbol{j} + m\boldsymbol{k}$, where m is a constant. Given that the lengths of $\overrightarrow{PQ}$ and $\overrightarrow{PC}$ are equal. Find the possible values of m.

23. The position vectors of points A and B are $\begin{pmatrix} -3 \\ 6 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$ respectively, relative to the origin O.

    a) Calculate angle AOB.

    b) The point C is such that $AC = 3AB$, find the ***unit position vector*** in the direction of $\overrightarrow{OC}$.

24. a) Find the equation of the line through points A and B with position vectors $\boldsymbol{3i - j + 4k}$ and $\boldsymbol{-4k + j}$

    b) A point C has position vector $\boldsymbol{6i + 4j + 5k}$. Find the perpendicular distance of C from the line in a) above

    c) The position vectors of three points A, B and C are p, $3q - p$ and $9q - 5p$ respectively. Show that the points are collinear.

25. Find the cartesian equation of a line that passes through three points; A(-2,0,4), B(5,2,6) and C(3,-1,-4).

    (**<u>Hint:</u> For three collinear points,** $\boldsymbol{r} = \overrightarrow{OA} + \beta\overrightarrow{BC}$).

26. The line l has equation: $\boldsymbol{r} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} + \beta\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ and point P has position vector $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

    a) Show that P does not lie on the line l

b) Given that a circle with centre P intersects l at points A and B, and A has position vector $\begin{pmatrix} 0 \\ -3 \\ 4 \end{pmatrix}$. Find the position vector of B. ***Ans:*** B(0,-3,6) OR B(-3,3,3)

27. Given the lines:

$l_1: \boldsymbol{r}_1 = -\boldsymbol{k} + \boldsymbol{i} + 2\boldsymbol{j} + \gamma(4\boldsymbol{j} + \boldsymbol{5i} + 3\boldsymbol{k})$ and $l_2: \boldsymbol{r}_2 = 3\beta\boldsymbol{k} + 2\boldsymbol{i} + 4\beta\boldsymbol{j} + \boldsymbol{k} + 5\beta\boldsymbol{i}.$

Show that;

a) l_1 and l_2 are parallel.

b) Perpendicular distance between l_1 and l_2 is $\frac{21\sqrt{2}}{10}$ units.

28. The line equation l_o has equation: $\frac{x-1}{2} = \frac{1-y}{2} = \frac{z+3}{-1}$ and the point A has co-ordinates (1,-2,1). Find the cartesian equation of the line that is perpendicular to l_o and passes through A. ***Ans***: $\left(\frac{x-1}{8} = y - 2 = \frac{z+1}{14}\right)$

29. The points A and B have position vectors $2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}$ and $4\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ respectively. The line l has equation: $\boldsymbol{r} = 4\boldsymbol{i} + 6\boldsymbol{j} + t(\boldsymbol{i} + 2\boldsymbol{j} - 2\boldsymbol{k})$. Point J lies on the line l such that $P\hat{A}B = 120^0$. Show that:

a) $3t^2 + 8t + 4 = 0$

b) Position vector of J is $2\boldsymbol{i} + 2\boldsymbol{j} + 4\boldsymbol{k}.$

30. The straight line l_1 passes through the points A(2,5,9) and B(6,0,10) and another line l_2 has equation: $\boldsymbol{r} = \begin{pmatrix} 8 \\ 8 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$.

a) Show that Point A is the intersection of l_1 and l_2.

b) Show further that l_1 and l_2 are perpendicular to each other

31. The points with co-ordinates A(7,6,10), B(6,5,6) and C(1,0,4) are the vertices of a parallelogram ABCD

a) Find the;

i) Co-ordinates of D

ii) Vector equation of the line l that passes through points A and C.

b) Show that the;

i) Shortest distance of l from B is $\sqrt{6}$ units.

ii) Exact area of a parallelogram ABCD is $18\sqrt{2}$ square units

32. Given the lines: $l_1: \boldsymbol{r} = \binom{2}{5} + \beta\binom{2}{-3}$ and $l_2: \boldsymbol{r} = \binom{3}{-3} + \mu\binom{3}{2}$.

a) Show that l_1 and l_2 are perpendicular.

b) Find the position vector of their point of intersection.

33. The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are such that $|\boldsymbol{a}| = 3$, $|\boldsymbol{b}| = 12$ and $\boldsymbol{a}.\boldsymbol{b} = 18$. Show clearly that; $|\boldsymbol{a} - \boldsymbol{b}| = 3\sqrt{13}$.

34. Relative to a fixed origin O, the straight lines l and m have vector equations; $\boldsymbol{r}_1 = \begin{pmatrix} p \\ 4 \\ 5 \end{pmatrix} + \gamma\begin{pmatrix} q \\ -1 \\ 2 \end{pmatrix}$ and $\boldsymbol{r}_2 = \begin{pmatrix} 9 \\ 0 \\ 16 \end{pmatrix} + \beta\begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix}$ where γ and β are scalar parameters, p and q are scalar constants. The point A is the intersection of l and m, and the cosine of acute angle θ between l and m is $\frac{1}{3}\sqrt{6}$

a) Find the values of p and q, given that q is a positive integer.

b) Determine the co-ordinates of A.

The points B has co-ordinates B(12,5,9)

c) Show that the;

i) Cosine of the acute angle φ between A and line l is $\frac{1}{3}$.

ii) $\varphi = 2\theta$.

Ans: $(q = 2,\ p = 0\ and\ A(8,2,9))$

35. The point A has position vector $-\boldsymbol{i} + 7\boldsymbol{j} - \boldsymbol{k}$.

a) Find the vector equation of the straight line l_1 which passes through A and parallel to the vector $3\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}$.

The straight line l_2 has equation: $\boldsymbol{r}_2 = 9\boldsymbol{i} - 9\boldsymbol{j} + 8\boldsymbol{k} + \beta(3\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k})$. Show that;

b) l_1 and l_2 do not intersect, and find the least distance between them.

c) Vector $2\boldsymbol{i} + 6\boldsymbol{j} + 3\boldsymbol{k}$ is perpendicular to both l_1 and l_2.

d) Point P lies on l_1 and M lies on l_2 so that $\overline{PM}$ is least, find the co-ordinates of P and M.

Ans: $[\,P(5,3,3) \quad and \quad M(3,-3,0)]$

36. The straight lines l_1 and l_2 have the following vector equations:

$\boldsymbol{r}_1 = \boldsymbol{i} - 5\boldsymbol{j} + \alpha(4\boldsymbol{j} - \boldsymbol{k})$ and $\boldsymbol{r}_2 = 4\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k} + \beta(3\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k})$ where α and β are scalar parameters.

a) Given that l_1 and l_2 intersect at point Q. Find the position vector of Q.

b) Given that P lies on l_1 and has position vector $\boldsymbol{i} + p\boldsymbol{j} - 3\boldsymbol{k}$**.** Find the value of p. (***Ans:*** $p = 7$)

c) The point T lies on l_2 so that $|\overrightarrow{PQ}| = |\overrightarrow{QT}|$, determine the possible position vectors of T. (**Ans:** $-5\boldsymbol{i} + 3\boldsymbol{j} - 5\boldsymbol{k}$, $7\boldsymbol{i} - 5\boldsymbol{j} + 3\boldsymbol{k}$)

37. Determine whether the following pairs of lines are parallel, concurrent (coincident) or skew: $\boldsymbol{r}_1 \equiv \begin{cases} x = 4 + 5t \\ y = 3 + 2t \\ z = 3t \end{cases}$ and $\boldsymbol{r}_2 \equiv \begin{cases} x = -5 + 2\beta \\ y = 4 - \beta \\ z = 1 \end{cases}$

38. The straight lines l_1 and l_2 have respective vector equations;

$\boldsymbol{r}_1 = (2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}) + \mu(\boldsymbol{j} + 3\boldsymbol{k})$ and $\boldsymbol{r}_2 = (\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) + \beta(\boldsymbol{i} + 2\boldsymbol{k})$ where μ and β are scalar parameters. Show that;

a) l_1 and l_2 are skew

b) Shortest distance between l_1 and l_2 is $\frac{5}{\sqrt{14}}$ Units using;

i) Dot product

ii) Cross product

39. Find the area of a triangle whose adjacent sides are represented by vectors; $\boldsymbol{2i - 3j + 5k}$ and $\boldsymbol{-k + 4j}$

40. Calculate the area of a parallelogram whose adjacent sides are represented by vectors; $\boldsymbol{3i - k}$ and $\boldsymbol{i - 4j + k}$**.**

41. Lines l_1, l_2 and l_3 are given by the equations:

$\frac{x+5}{2} = \frac{y-14}{-10} = \frac{z+13}{11} = \alpha$, $\frac{x-3}{2} = \frac{y+5}{-3} = \frac{z+17}{-5} = \beta$ and $x = y + 5 = \frac{z-7}{-2} = \gamma$ respectively. Given that l_1 intersects l_2 at A while l_1 intersects l_3 at B, find the distance AB.

42. Find the equation of the line perpendicular to the plane $3x - 7y + 2z = 8$ and passing through a point with position vector $\boldsymbol{-5j + k - i}$.

43. Show that the least distance of A(0,3,5) from the line $\boldsymbol{r} = \boldsymbol{2i} + \beta\boldsymbol{k} - \boldsymbol{j} - \beta\boldsymbol{i} + 3\boldsymbol{k} + 2\beta\boldsymbol{j}$ is zero.

44. Find the angle between vector $\boldsymbol{3k - 4j}$ and the line $\boldsymbol{r} = -\beta\boldsymbol{j}$.

PLANES

45. Find the equation of the plane that contains points; (1,3,2), (2,1,-1) and (2,4,1) in;

a) Scalar,

b) Cartesian, form.

46. Find the cartesian equation of the plane that contains a point whose position vector is $\boldsymbol{i - 4k + 2j}$ and normal to the vector $\boldsymbol{3j + k - 4i}$.

47. Two lines; l_1 and l_2 have equations; $\boldsymbol{r}_1 = \begin{pmatrix} 5 \\ 1 \\ -4 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ and $\boldsymbol{r}_2 = \begin{pmatrix} p \\ 4 \\ -2 \end{pmatrix} + t \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix}$ respectively, and p is a constant. If l_1 and l_2 intersect at Q, find the;

i) Value of p

ii) Position vector of Q

iii) Equation of the plane containing l_1 and l_2.

48. A point M has position vector $\boldsymbol{3i - 2j + k}$. The line l has equation $\boldsymbol{r} = \boldsymbol{4i + 2j + 5k} + \gamma(\boldsymbol{i + 2j + 3k})$. Find the cartesian equation of the plane containing l and M.

(***Ans***: $4x + y - 2z = 8$)

49. Given the equation of the plane, π: $\boldsymbol{r} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix}$. Find the;

a) Normal vector to π

b) Cartesian equation of π

c) Angle with which π makes with the line l: $\frac{1-x}{2}=\frac{y+2}{1}=\frac{z+3}{4}$. Hence, find the co-ordinates of the point of contact.

50. Find the equation of the plane that contains points whose position vectors are respectively; $\boldsymbol{2i-3k}$ and $\boldsymbol{-j+i-4k}$.

51. A line l is given by the vector equation: $\boldsymbol{r}=\begin{pmatrix}\boldsymbol{3}\\\boldsymbol{-1}\\\boldsymbol{4}\end{pmatrix}+\beta\begin{pmatrix}-2\\3\\1\end{pmatrix}$ and a point N has a position vector $\boldsymbol{3i-j+4k}$.

a) Find the cartesian equation of a plane π_1 that contains l and N.

b) Given that a line l_0 passes through a point with position vector $\boldsymbol{-6k+2i-j}$ and is parallel to π. Find the cartesian equation of l_0.

c) Hence, find the least distance between l and l_0.

52. The planes π_1 and π_2 have equations: $3x+y-2z=10$ and $x-2y+2z=5$ respectively. The line l has equation $\boldsymbol{r=4i+2j+k}+\beta(\boldsymbol{i+j+2k})$.

a) Show that l is parallel to π_1

b) Calculate the acute angle between π_1 and π_2

c) Point P lies on l, the perpendicular distance of P from the plane π_2 is equal to 2. Find the two position vectors of P.

53. Three points (-3,-6,11), (2,-1,6) and (6,0,-10) lie on the plane π.

a) Find the cartesian equation of π

b) Verify that point (7,2,-7) lies on the plane π.

54. a) Find the vector equation of the line passing through the point (3,1,2) and perpendicular to the plane $\boldsymbol{r}.(\boldsymbol{2i-j+k})=4$. Hence, find the point of intersection of the line and the plane.

c) The position vectors of the points A, B and C are $\boldsymbol{2i-j+5k}$, $\boldsymbol{i-2j+k}$ and $\boldsymbol{3i+j-2k}$ respectively. Given that L and M are midpoints of AC and CB respectively. Show that $BA=2ML$

55. Show that P(1,2,-1) and Q(3,6,1) lie on opposite sides of the plane; $\boldsymbol{x+y+4z=5}$

56. Find the cartesian equation of the plane through the points A(1,0,-2) and B(3,-1,1) which is parallel to the line with vector equation $\boldsymbol{r} = \boldsymbol{3i} + (2t-1)\boldsymbol{j} + (5-t)\boldsymbol{k}.$ Hence, find the co-ordinates of the point of intersection of this plane and the line $\boldsymbol{r} = \mu\boldsymbol{i} + (5-\mu)\boldsymbol{j} + (2\mu - 7)\boldsymbol{k}.$

57. A plane contains points A(4,-6,5) and B(2,0,1). A perpendicular to the plane from the point P(0,4,-7) intersects the plane at point C. find the co-ordinates of C.

58. Find the equation of a plane parallel to the vectors $\boldsymbol{r}_1 = \boldsymbol{i} + \boldsymbol{3j} + \boldsymbol{5k}$ and $\boldsymbol{r}_2 = \boldsymbol{4i} - \boldsymbol{j} + \boldsymbol{2k}$ which passes through a point with position vector $\boldsymbol{2i} + \boldsymbol{3k}.$

59. a) If M is the foot of a perpendicular from point A(5,-3,2) to the line $\frac{x-1}{2} = \frac{y-4}{1} = \frac{z-2}{-1}$. Determine the co-ordinates of M.

b) Find the equation of the plane through points M, A and B(-1,4,2) on the line. Hence, determine the angle ABM.

60. l is a line: $\frac{x}{2} = \frac{y-1}{1} = \frac{z+1}{-1}$ and π is the plane: $x - 4y - 2z = 5$.

a) Show that l is parallel to π, and calculate the distance between l and π

b) Find the cartesian equation of a second plane π_0 which contains l and is perpendicular to π.

61. Determine the cartesian equation of the plane that is parallel to the line with equation $x = -2y = 3z$ and contains the line of intersection of two planes: $x - y + z = 1$ and $2y - z = 0$ (***Ans:*** $8x + 14y - 3z = 8$)

62. A plane π passes through point A(3,1,4).

Find the cartesian equation of π that passes through the intersection of two planes: $x + 2y + 3z = 1$ and $2x - y + z = -3$. (***Ans:*** $x - 2y - z + 3 = 0$).

63. Find the co-ordinates of the point of intersection of the line: $x = \frac{y+1}{2} = \frac{z+1}{3}$ and the plane that passes through A(4,0,1), B(0,-3,0) and C(6,3,3).

64. Show that (2,4,1) lies on the plane that contains points; (4,2,3), (5,1,4) and (-2,1,1).

65. Find the equation of the plane that passes through points (1,1,0), (4,-2,1) and is perpendicular to the plane $3x + y - z = 2$. (**Ans:** $x + 3y + 6z = 4$)

66. Find the equation of the plane containing a line: $\frac{1-x}{-2} = \frac{1-y}{-1} = \frac{z+2}{-1}$ and is perpendicular to the plane $x - 2y + 3z = 1$. (**Ans:** $x - 7y - 5z = 4$).

67. Show that the lines: $\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z+2}{4}$ and $\boldsymbol{r} = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} + \gamma \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}$ are parallel to each other.

Find the cartesian equation of the plane containing the two lines.

68. A line l and a plane π have the following equations:

l: $\frac{x+1}{2} = \frac{y-3}{5} = \frac{z+2}{-1}$ and $\pi: x + y + z = 12$. Given that l and π meet at point A.

a) Find the position vector of A

b) Calculate the angle between l and π.

c) Find the cartesian equation of a plane π_1 that;

i) contains l and a point whose position vector is $\boldsymbol{2i - 3j + k}$.

ii) Is parallel to plane π and contains a point A(1,-2,3)

iii) Is perpendicular to plane π and contains l.

69. Line l passes through R(3,1,-2) and S(4,-1,2). A plane OMN where O is the origin, and M and N have position vectors $\boldsymbol{3j}$ and $\boldsymbol{i + 2j}$ respectively meets l at A.

Find the co-ordinates of l.

70. A vector $\boldsymbol{-j + k - 3i}$ is parallel to the plane that contains two points A(1,1,2) and B(3,0,4).

a) Write down the vector equation of the plane. Hence, or otherwise, find the cartesian equation of the plane.

b) Calculate the distance of the point C(5,4,-5) from the plane in a) above.

71. The distance of the point A(4,-1,2) from the plane is $\sqrt{3}$ units. Given that the vector $\boldsymbol{i + j + k}$ is normal to the plane, find the cartesian equation of the plane.

72. Find the equation of a plane through a point (3,-6,-7) and orthogonal to the line parametrically given by;

$$\boldsymbol{r} = \begin{cases} x = 2 + 3t \\ y = 1 + 4t \\ z = 7 - 8t \end{cases}.$$

73. Find the equation of the plane π containing the following lines;

a) $\frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}$ and $\frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$.

b) $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-6}{6}$.

c) $\boldsymbol{r}_1 = t\boldsymbol{i} + (3t-6)\boldsymbol{j} + (t+1)\boldsymbol{k}$ and $\boldsymbol{r}_2 = (2t-4)\boldsymbol{i} + (t-3)\boldsymbol{j} + \boldsymbol{k}$.

d) $\frac{x-1}{-4} = y = z+1$ and $\frac{x+1}{-4} = y-1 = z-3$.

e) $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x}{4} = \frac{y-2}{-2} = \frac{z+1}{6}$. Also, find whether the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{5}$ lies on the plane.

74. The co-ordinates of the points A and B are (0,2,5) and (-1,3,1) respectively, and the line l has equation: $\frac{x-2}{2} = \frac{y-2}{-2} = \frac{z-2}{-1}$.

a) Find the equation of the plane π which contains point A and is perpendicular to l, and verify that point B lies on π

b) Show that point C at which line l meets π is (1,4,3), and find the angle between CA and CB.

75. Find the co-ordinates of point on the line whose equation: $x = \frac{y+1}{2} = \frac{z+1}{3}$ which is 2 Units from a plane which contains points A(4,0,1), B(0,-3,2) and C(6,3,3).

76. Given the equations of three planes: $2x - y + 3z = 4$, $3x - 2y + 6z = 3$ and $7x - 4y + 5z = 11$. Find their point of intersection.

77. The equations of the two planes π_1 and π_2 are;

$$\boldsymbol{r}_1 = \begin{pmatrix} \mathbf{1} \\ \mathbf{0} \\ \mathbf{1} \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -2 \\ \mathbf{2} \end{pmatrix} + \alpha \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} \text{ and } \boldsymbol{r}_2 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + t \begin{pmatrix} 4 \\ 0 \\ 5 \end{pmatrix} + \rho \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}.$$

a) Find the line l of intersection of π_1 and π_2

b) Find the cosine of the acute angle between π_1 and π_2

c) Show that the length of the perpendicular from the point (1,5,1) to l is $\sqrt{2}$ Units.

d) Calculate the angle between π_1 and π_2.

78. Find the shortest distance from the origin to the plane $3x - 4y - z + 26 = 0$

79. Find the cartesian equation of the plane π_0 that contains a point whose position vector is $\boldsymbol{i} - \boldsymbol{j}$ and is orthogonal to the line l: $\boldsymbol{r} = \begin{cases} x = 3 + 2t \\ z = -t + 3 \\ y = 1 + 3t \end{cases}$.

80. Find the perpendicular distance of the point (2,1,0) from the plane $-x - 5y + 2z = 9$

81. Find the distance of the point (-1,-5,-10) from the point of intersection of the plane

$x + 3y - 2z = 3$ and the line $\frac{x-1}{3} = \frac{1-z}{-4} = y - 2$.

82. Find the position vector of the point of intersection of the line: $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane: $x - y + z = 16$.

83. Points A, B and C are such that $OA = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$, $OB = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$ and $OC = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$. Find the equation of the plane π perpendicular to AB and contains point C.

84. The point M has co-ordinates (2,0,-1) and the plane π has the equation $x + 2y - 2z = 8$. The line through M parallel to line l: $\frac{x}{2} = y = \frac{z+1}{1}$ meets π at point C.

a) find the co-ordinates of C.

b) find the length of MC

85. Given that lines: l_1: $\boldsymbol{r} = \boldsymbol{i} + \boldsymbol{2k} + t(2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k})$ and $\boldsymbol{r} = -\boldsymbol{2i} + \boldsymbol{2j} + \boldsymbol{k} + \beta(-\boldsymbol{3i} + \boldsymbol{2j} - \boldsymbol{k})$ intersect at A.

a) Find the co-ordinates of A

b) Cartesian equation of the plane π_0 containing l_1 and l_2.

c) Cartesian equation of the plane π that contains a point J(3,-2,1) and is parallel to π_0

d) Symmetric equation of the line l that is normal to π_0 and passes through A(0,-1,1)

86. Find the co-ordinates of the point of intersection of lines:

l_1: $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and l_2: $\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3}$. Hence, find the;

a) Equation of the plane π containing l_1 and l_2 in the form $\boldsymbol{r}.\boldsymbol{n} = D$

b) Vector equation of the line l_0 normal to both l_1 and l_2 and passes through B(1,2,-4)

c) Cartesian equation of the plane π_1 that contains A(5,-1,2) and l_0

d) Perpendicular distance of π_1 from the origin.

87. A plane π contains a point whose position vector is $\boldsymbol{i} - 2\boldsymbol{j} - \boldsymbol{k}$ and is parallel to two lines l_1 and l_2 parametrically given as:

$$\boldsymbol{r}_1 = \begin{cases} x = 2 + \beta \\ y = 3\beta - 4 \\ z = 1 - 2\beta \end{cases} \quad \text{and} \quad \boldsymbol{r}_2 = \begin{cases} z = 6\gamma + 1 \\ x = -3 - 9\gamma \\ y = -3\gamma \end{cases}.$$

a) Write down the general vector equation of π.

b) Find the;

i) Normal vector of π

ii) Cartesian equation of π

iii) Shortest distance of l_1 from π .

88. Given that $\boldsymbol{a} = 3\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = -\boldsymbol{j} + 2\boldsymbol{k} + 3\boldsymbol{i}$. Find the;

a) Angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

b) Unit vector normal to both $\boldsymbol{a}$ and $\boldsymbol{b}$

c) Plane π that contains $\boldsymbol{a}$ and $\boldsymbol{b}$

d) Line l that passes through points A and B with above position vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ respectively

e) Plane π_0 that is parallel to π, and contains point A(3,-1,0)

f) Plane π_1 that is normal to π and contains l.

89. Find the parametric equations of the line of intersection of planes π_1 and π_2

$\pi_1: x + 2y - 3z = 4$ and $\pi_2: 3x - 4y + z = -3$

90. Using the general cartesian equation of the plane: $ax + by + cz + d = 0$, find the equations of planes that contain the following points;

a) (2,3,1), (1,-1,2) and (2,1,-2)

b) (0,0,2), (1,2,3) and (-2,0,-1)

91. Find the angle between the line: $\frac{x+2}{2} = \frac{1-z}{-1} = \frac{1+y}{3}$ and the plane; $3x + y + z = 1$

92. Show that the point (1,2,-3) lie on the plane $2x + y - z = 7$.

93. A plane π contains points; (1,2,5), (1,0,4) and (5,2,1). Show whether or not a point with position vector $-4\boldsymbol{k} + \boldsymbol{2i} + \boldsymbol{j}$ lies on π.

94. Find the distance between planes π_1 and π_2

$\pi_1: x + y - z + 2 = 0$

$\pi_2: x + y - z + 6 = 0$

95. The plane π has vector equation: $\boldsymbol{r} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \beta \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} + \rho \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$. Show that the point with position vector $\begin{pmatrix} 2 \\ 2 \\ -7 \end{pmatrix}$ lies on the plane π.

96. Find the perpendicular distance from the plane $\boldsymbol{r}.(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{2k}) = 4$ to the origin.

97. Find the co-ordinates of the foot of the perpendicular drawn from point A(1,0,-1) to the;

a) Plane: $\boldsymbol{r}.(\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}) = 6$

b) Line: $\frac{x+1}{2} = \frac{y-4}{-1} = \frac{z+2}{3}$

98. Prove that (0,0,0) and (2,-3,7) lie on the same side of the plane: $2x - 3y + 2z + 8 = 0$.

99. Find the cartesian equation of the plane that contains point (1,1,-2) and normal to the vector $\boldsymbol{2i - 3j + 4k}$**.**

100. A plane $\pi: 2x - 3y + z = 5$ and the line $l: \boldsymbol{r} = \begin{cases} y = 3 - t \\ z = 2 + 3t \\ x = 1 + 2t \end{cases}$ intersect at R. Find the;

a) ***Position vector*** of R

b) Angle between π and l

c) Distance of l from the origin

d) Equation of the plane π_0 that contains l and parallel to vector $\boldsymbol{i + 2j + 3k}$**.**

END